SPE 75703

Why Plot the Equivalent Time Derivative on Shut-in Time Coordinates?

David Anderson, Fekete Assoc. Inc., Colin Lyle Jordan, Fekete Assoc. Inc., Louis Mattar, Fekete Assoc. Inc.

This paper was prepared for presentation at the SPE Gas Technology Symposium held in Calgary, Alberta, Canada, 30 April–2 May 2002.

ABSTRACT

The (semilog) derivative of pressure data from oil and gas wells is a widely accepted and highly useful tool for reservoir characterization. For buildup pressure data, this derivative is calculated with respect to equivalent time. It is generally assumed that since this derivative is obtained using equivalent time, it should be plotted on a coordinate axis that uses equivalent time. However, in the presence of boundaries, this style of plotting distorts the shape of the derivative during late-time, to the point where it bears very little resemblance to the original shape of the drawdown derivative. Moreover, the shape of the buildup derivative is strongly affected by the duration of the preceding drawdown.

This paper investigates the shape of the equivalent-time-derivative when plotted against both equivalent time and real time (shut-in time) coordinates. Using computer generated models, it shows that a plot based on real time produces a buildup derivative response that is much closer in shape to the drawdown derivative, thus resulting in more accurate diagnosis of reservoir characteristics. This was true regardless of reservoir type, geometry or producing time. The advantages of using real time as a plotting coordinate are, a more readable diagnostic plot and a more easily recognized shape because of its similarity to the drawdown.

Our research extends the work of Spivey (1999). This study investigated numerous reservoir types including homogeneous with boundaries, composite, fractured and dual porosity, and it also covered a wide practical range of producing times.

INTRODUCTION

A great deal of literature exists on the use of derivative typecurves for buildup tests. The general consensus in the literature is that a buildup pressure derivative, calculated with respect to equivalent time, should be plotted against equivalent time to give a shape that is similar to the drawdown derivative. This procedure works provided that the radial flow assumption is valid and producing time is sufficiently long. However, in cases where boundaries, heterogeneities, or fracture flow exist in conjunction with short production times, the opinions of some experts vary.

Spivey et al (1999) suggest plotting the buildup pressure derivative calculated with respect to shut-in time against shut-in time for all reservoir cases[1]. Although this method allows the welltest analyst to develop a complete set of buildup type-curves that do not rely on the pre-assumption of radial flow, the resulting curves bear little resemblance to their drawdown counterpart for many reservoir configurations. More importantly, the shapes of these typecurves depend on producing time, thereby adding to the complexity of the diagnostic analysis.

Onur and Satman (1998) suggest using the conventional plotting method (equivalent time derivative plotted against equivalent time) for all cases, except when producing time is short, in which case they propose to plot the equivalent time derivative against shut-in time [2]. Although this method provides buildup type curves that are similar in shape to drawdown type curves, it requires the analyst to decide what is a "short" producing time. Indeed, an experienced welltest analyst can usually identify the effects of a "too short" production time (or a significant rate change that occurs very close to shut-in) on a plot of equivalent time buildup derivative against equivalent time. Upon identification of these equivalent time effects, the analyst would then re-plot the equivalent time buildup derivative against real shut-in time.

The method we propose eliminates the need for the analyst to be able to identify producing time effects from the diagnostic plot, but preserves a set of buildup typecurves that are very similar in shape to drawdown typecurves for a wide variety of reservoir configurations. The method is simple: always plot the equivalent time buildup derivative against shut-in time, regardless of the duration of the preceding drawdown period.

METHODOLOGY

In this study, two different methods of plotting the equivalent time buildup derivative were evaluated. The research was carried out by generating synthetic flow and buildup pressure data, and then comparing the signature of the equivalent time buildup derivative, when plotted against equivalent time and against real time, to the signature of the corresponding drawdown derivative. The synthetic data was generated using a variety of models, including 1) homogeneous reservoirs with boundaries, 2) radial composite reservoirs, 3) hydraulically fractured reservoirs, and 4) dual porosity (naturally fractured) reservoirs. The scope of this work focuses on transient flow behavior, as this is the primarily domain of welltest analysis. For each model investigated, three derivative plots were generated as listed below:

1) Drawdown derivative vs. drawdown time.
2) Equivalent time buildup derivative vs. equivalent time. (to be refered to as "equivalent time derivative")
3) Equivalent time buildup derivative vs. real time. (to be refered to as "real time derivative")

The real time derivative term used in this paper should not be confused with the pressure derivative taken with respect to shut-in time ($\partial P/\partial(\ln \Delta t)$).

For each model, three different producing times were investigated: short (10 hrs), medium (100 hrs) and long (1000 hrs). In addition, two different reservoir permeabilities were investigated for each model: low (1 mD) and high (100 mD). In each case, the buildup derivative was plotted against both real time and equivalent time, and compared to the drawdown derivative in terms of similarity of shape. The results are discussed in the "Synthetic Examples" section of the report. In addition to the synthetic cases, two field examples are presented, in an effort to illustrate the practical applicability of the results.

Furthermore, in Appendix A, we attempt to confirm our experimental observations through the analytical formulation of the buildup and drawdown derivatives. In this paper, we focus our analytical work on a scenario with a well near a single boundary.

EQUIVALENT TIME

Equivalent time is defined as follows[3]:

$$t_e = \frac{\Delta t \cdot t_p}{\Delta t + t_p} \tag{1}$$

Equation (1) implies that t_e is always less than t_p (refer to Figure 7). Therefore, for shut-in times exceeding the producing time, t_e appears compressred relative to Δt when both are plotted on the same scale. This phenomenon will be referred to as the compression effect.

RESULTS

The results of the study are as follows:

1) The real time derivative always maintains the basic shape of the corresponding drawdown derivative, regardless of producing time, reservoir characteristics or flow geometry.
2) The equivalent time derivative is highly sensitive to changes in producing time and reservoir permeability, and only conforms to the shape of the drawdown derivative when these parameters are sufficiently large, or if shut-in time is short relative to the flow period.
3) The potential for misinterpretation of the buildup derivative when plotted against equivalent time increases when the compression effect occurs in the vicinity of a reservoir anomaly such as a boundary or heterogeneity.

Several simulation cases are presented in this paper as verification of the wide range of applicability of plotting the pressure derivative against shut-in time coordinates.

SYNTHETIC DATA EXAMPLES

In order to evaluate the impact of plotting the buildup derivative against real time, a number of producing reservoir scenarios were investigated. The most significant cases are described below, and the results are plotted in Figures 1a through 4c. The corresponding drawdown derivative (with a producing time of 1,000 hours) has been superimposed on all plots for comparison.

CASE #1 –RECTANGULAR HOMOGENOUS RESERVOIR. In this set of scenarios, we investigate the effect of no-flow boundaries on the behavior of the equivalent time and real time derivatives.

Case 1a –Centered Well in Closed Reservoir; Low Permeability (1 mD). This scenario may be considered the "base case" as it is the simplest flow geometry investigated in this paper. Following wellbore storage, the shape of the derivative should be horizontal (during radial flow), followed by a sharp falling off due to pressure stabilization. Figure 1a compares the shape of the real time and equivalent time buildup derivatives to the corresponding drawdown derivative. From the plot, it is clear that the radial flow period is compressed on the equivalent time plot, when producing time is short.

In contrast, the real time derivative maintains its shape regardless of the production time. However, due to the simplicity of the flow geometry, the compression effect of the radial flow period on the equivalent time derivative does not interfere with accurate interpretation of the transient response.

Case 1b –Centered Well in Closed Reservoir; High Permeability (100 mD). Figure 1b illustrates the results of this scenario. Although the equivalent time derivative exhibits the same compression effect observed during the radial flow period in Case 1a, the degree of compression is significantly less in this scenario. Thus, it is evident that increasing permeability has a negative effect on the apparent amount of distortion apparent on the equivalent time derivative. On the other hand, the real time derivative maintained its uniformity with the drawdown derivative regardless of reservoir permeability

*Case 1c – Well Near One Boundary in a Closed Reservoir; High Permeability (100 mD).*In this scenario, the presence of the boundary complicates the shape of the derivative. Specifically, after radial flow, there should be a doubling of the derivative, followed by a falling off due to pressure stabilization.

The results of this case are shown in Figure 1c. From the plot, it is evident that the boundary is observed before any compression effects begin to distort the equivalent time derivative, therefore the data can still be accurately interpreted. The severity of the compression appears to be very similar to that of Case 1b. Thus, it appears that it is a combination of permeability and flow geometry that influences how well the equivalent time derivative will conform to the drawdown derivative. In order to confirm these results, Case 1c was re-run but with a lower reservoir permeability (refer to Case 1d).

Case 1d – Well Near One Boundary in a Closed Reservoir; Low Permeability (1 mD). As anticipated, the low permeability case creates much more distortion in the equivalent time derivative since the boundary is observed during the late-time when compression effects are the greatest. Figure 1d shows that the hemi-radial flow period following the appearance of the boundary is significantly compressed on the equivalent time derivative plot (for a producing time of 10 hours). As in previous cases, the real time derivative remains unaffected by the change in permeability and continues to reasonably follow the shape of the drawdown derivative.

Additional Cases- Well Near Boundaries. In order to confirm previous observations for a wider range of flow situations, scenarios featuring a well near two and three boundaries were also investigated.

For a high permeability scenario, the results for a well near two or three boundaries were identical to that of Case 1c. This confirms the earlier observations that producing time, permeability and flow geometry determine the degree of distortion in the equivalent time derivative. Figure 1e shows the results of a high permeability case (100 mD) with a well near two boundaries.

For a low permeability scenario, the compression of the equivalent time derivative occurs very soon after the appearance of the first boundary, thus masking the remainder of the reservoir response. In contrast, the real time derivative clearly identifies all three boundaries. Figure 1f illustrates a low permeability, low producing time scenario featuring a well near three boundaries.

General Observations and Conclusions – Case 1

- The real time derivative always maintains the basic shape of the drawdown derivative, regardless of 1) production time, 2) permeability or 3) number of boundaries.
- The equivalent time derivative exhibits compression effects when either producing time or permeability is low. In some situations, the compression effects can significantly influence the analyst's interpretation of the transient response (eg. the proper identification of hemi-radial flow following the observation of a single boundary).

CASE #2 – SYMMETRICAL COMPOSITE RESERVOIR. After investigating the effects of boundaries, we now explore the effects of reservoir heterogeneities on the behavior of the equivalent time and real time derivatives.

Case 2a – High Permeability Inner Zone (100 mD); Low Permeability Outer Zone (0.1 mD). This scenario simulates an abrupt radial decrease in permeability at a short distance from the well. This type of reservoir heterogeneity ought to appear similar to a single no-flow boundary on the buildup derivative. Since this reservoir system is characterized by lower permeability, we would expect similar compression effects (like those observed in Case 1) on the equivalent time derivative.

As expected, Figure 2a shows significant compression on the equivalent time derivative, while the real time derivative maintains the character of the drawdown derivative. In this case, the radial flow regime of the outer zone is completely distorted on the equivalent time derivative. Thus, accurate diagnostic analysis, under these conditions, would not be possible if the buildup derivative were plotted against equivalent time.

Case 2b - High Permeability Inner Zone (100 mD); Low Permeability Outer Zone (10 mD). For comparison, the composite reservoir simulation was re-run but the permeability of the outer zone was increased to 10 mD.

Figure 2b illustrates that the compression effect on the equivalent time derivative is greatly reduced (as the outer reservoir permeability is larger than before) and no longer impedes the accurate interpretation of the transient response. At higher outer region permeabilty, the transition between zones is observed before compression effects begin to distort the data. As expected, the real time derivative retains its character.

Case 2c – Lower Permeability Inner Zone (0.1 mD); Higher Permeability Outer Zone (100 mD). This scenario is essentially the same as the high permeability homogeneous system. The results from these simulations (not included in this report) are consistent with the high permeability scenarios evaluated in Case1.

General Observations and Conclusions – Case 2

- The real time derivative always maintains the basic shape of the drawdown derivative, regardless of 1) production time, 2) permeability or 3) outer zone permeability.
- The equivalent time derivative exhibits compression effects when either producing time or outer zone permeability is low. In some scenarios, the compression of the equivalent time derivative completely masks the second radial flow period.

CASE # 3 – CIRCULAR RESERVOIR WITH A SINGLE HYDRAULIC FRACTURE. Case #3 is a departure from the previous two cases, as it represents a different type of flow geometry; namely, linear (or bilinear) fracture flow, transitioning into radial flow.

Case 3a – Low Permeability Reservoir- (1 mD, X_f = 100 m). The results of this scenario (Figure 3a) show that despite the change in flow geometry, the compression effect on the equivalent time derivative occurs in a manner similar to that of Cases 1 and 2. The radial flow period following the fracture flow dominated portion of the equivalent time derivative is severely distorted. In this case, the compression masks the true transient response even for producing times as high as 100 hours (an inexperienced analyst may misinterpret this late-time behavior as a boundary effect).

Again, the real time derivative maintains the same shape as the drawdown derivative, even for producing times as short as 10 hours. Additional cases (not included in this report) show that shorter fracture lengths, while not affecting the degree of compression, allows improved accuracy of the interpretation because the compression occurs at a later shut-in time (thus the transition into radial flow is not masked)

Case 3b – High Permeability Reservoir (100 mD, X_f = 100 m). As predicted for a high permeability system, the results of this case (Figure 3b) show that neither the equivalent time nor the real time derivative behavior deviates significantly from the drawdown derivative (refer to Figure 3b). Yet, at low producing times, the equivalent time derivative still yields the expected compressed radial flow period, which makes it more difficult to characterize the reservoir with confidence.

General Observations and Conclusions – Case 3

- The results of the hydraulic fracture scenarios are consistent with Cases 1 and 2. However, the potential for misinterpretation of the transient pressure response when using the equivalent time derivative appears to be greater.

CASE # 4 – DUAL POROSITY RESERVOIR. Finally, we investigate a naturally fractured reservoir model, which often exhibits a unique signature on the buildup (and drawdown) derivative.

Case 4a – Low Interporosity Flow Coefficient (matrix permeability = 1 mD, λ= 1E-6). The simulation for this case indicates that the equivalent time derivative deviates significantly from the drawdown derivative, even for producing times as high as 100 hours (refer to Figure 4a). As in the single fracture or composite reservoir cases, the late time of the equivalent time derivative is severely distorted, masking the radial flow regime. The buildup derivative, when plotted against real time, mimics the drawdown derivative shape, regardless of producing time or fracture properties (refer to Figure 4a.2). To confirm these results, the case was re-run but with a high interporosity coefficient. Incidentally, the simulation was not sensitive to changes in storativity ratio (for a range for 0.1 to 0.05).

Case 4b- High Interporosity Flow Coefficient (matrix permeability = 1 mD, λ = 1e-4). By increasing the interporosity flow coefficient, the dual porosity effect is observed before any compression effects occur (refer to Figure 4b. Compression effects occur during the radial flow period, and therefore do not significantly alter the interpretation of the derivative. The same results are achieved if matrix permeability is high, and the interporosity coefficent remains low.

General Observations and Conclusions – Case 4

- For naturally fractured reservoirs with a low interporosity coefficient, the real time buildup derivative tracks the drawdown derivative closely regardless of 1) matrix permeability, 2) storativity ratio , or 3) flow time.
- For naturally fractured reservoirs with a high reservoir permeability, the buildup derivative tracks the drawdown derivative closely regardless of 1) the time plotting function, 2) interporosity coefficient, 3) storativity ratio, or 4) producing time duration.

FIELD EXAMPLES

In an effort to illustrate the practical implications of the synthetic data research, two actual welltest examples are presented in the following section. These examples are plotted in Figure 5a through 6c.

Field Example #1. The rate and pressure data for this example are shown in Figure 5a. The well was flowed on clean-up for about 1 day, followed by shut-in. After about 2 days of shut-in (which was sufficient for pressure stabilization), the well was opened to flow for 10 hours. Then, the well was shut-in for a buildup test. The effective permeability to gas in the vicinity of this well was estimated to be less than 30 mD.

The original derivative analysis was performed by plotting the buildup derivative against equivalent time (refer to Figure 5b). The interpretation of the data suggests a brief period of radial flow, followed by the appearance of a geological feature such as a boundary or heterogeneity. Ultimately, this data was interpreted as a well being completed close to one or more no-flow boundaries.

In contrast, when we plot the derivative against real time (refer to Figure 5c), the impression of the data is much different. Here, the analyst could interpret that after wellbore storage, radial flow dominated all the data since there is no longer a sharp increase in the pressure derivative during the late-time. It is still possible that the slight rise in the derivative that occurs after 100 hours of shut-in is due to a nearby boundary. But given the scatter in the data set, the apparent late-time boundary effect may not be real. Thus, by plotting the buildup derivative against real time, the interpretation of the reservoir characteristics is completely different.

Field Example #2. The rate and pressure data for this example are shown in Figure 6a. As with Field Example #1, this data was taken from a flow and buildup test performed on the well just after completion. However, this well was also hydraulically fractured. The well was then flowed for approximately 40 hours before the buildup test. The effective permeability to gas in the vicinity of this well was estimated to be less than 1 mD.

Initially, the data was analyzed by plotting the buildup derivative against equivalent time (refer to Figure 6b). The plot shows that linear fracture flow appears to dominate the entire test, except during the last few hours of shut-in where the slope of the pressure derivative increases sharply. Radial flow does not appear to have developed during the test. An analyst may interpret this as an effective hydraulic fracture, followed by some reservoir anomaly.

However, when the same derivative data is plotted against real time (refer to Figure 6c), the physical interpretation of the data is much different. The late-time effect observed from the equivalent time derivative is not present, and has been replaced with well developed radial flow.

CONCLUSIONS

When plotting the buildup derivative against equivalent time, the degree of conformance to the drawdown derivative was found to be highly sensitive to permeability/flow geometery and producing time. For small producing times, the buildup derivative tends to have a compressed appearance when plotted against equivalent time. This compression effect often distorts the character of the derivative when reservoir anomalies such as boundaries or heterogeneities occur in the late-time. This distortation sometimes leads to misinterpretation of the derivative.

We have shown experimentally that a diagnostic plot featuring a buildup derivative graphed against real time coordinates always yields a typecurve that closely resembles the drawdown derivative (in the transient regime only), regardless of 1) reservoir characteristics, 2) flow geometry or 3) producing time.

The observations and conclusions from the simulation runs have been confirmed through the development of an analytical form of the buildup derivative for the simple case of a well near one boundary (refer to Appendix A). For this model, we have shown mathematically that as producing time approaches zero, the equivalent time derivative retains a form very similar to the drawdown derivative, provided the independent variable is shut-in time (not equivalent time). This analysis supports our experimental observations, which indicate that the buildup derivative maintains the shape of the drawdown even when producing times are very short.

In consideration of the above, we recommend that the equivalent time buildup derivative always be plotted against real time coordinates, regardless of the duration of the flow, shut-in times or reservoir flow parameters.

NOMENCLATURE

Δt	-	Shut-in or real time
Δt_D	-	Dimensionless shut-in time
t_e	-	Equivalent time, hr
t_{eD}	-	Dimensionless equivalent time
t_p	-	Producing time, hr
t_{pD}	-	Dimensionless producing time
x	-	Distance to boundary, m
r_w	-	wellbore radius, m
p	-	Pressure, kPa
p_D	-	Dimensionless pressure
Ei	-	Exponential integral function
f_1	-	Coefficent function for limit form of buildup derivative
f_2	-	Coefficent function for limit form of buildup derivative

REFERENCES

1. Spivey, John P. et al: "Application of the Diagnostic Plot Using a Derivative Based on Shut-in Time," SPE Paper 56424, presented at the 1999 SPE Annual Technical Conference and Exhibition, Houston, Texas, 3-6 October 1999.
2. Onur, M and Satman, A: "Interpretation of Single Well Pressure Transient Data From Naturally Fractured Reservoirs," Department of Petroleum and Natural Gas Engineering, Istanbul Technical University, Maslak, Istanbul, Turkey.
3. Agarwal, R.G.: "A New Method to Account for Producing Time Effects when Drawdown Type Curves are Used to Analyze Pressure Buildup and Other Test Data," SPE Paper 9289 presented at the 1980 SPE Annual Fall Technical Conference and Exhibition, Dallas, Texas, 21-24 September, 1980.

Appendix A – Theoretical Considerations- An Analytical Form of the Buildup Derivative

Experimental results show that plotting the buildup derivative against real shut-in time results in a derivative signature that is more consistent with the drawdown derivative, as opposed to the conventional method of plotting the buildup derivative against equivalent time.

The objective here is to show mathematically, that the buildup derivative corresponds more closely to its drawdown counterpart at low producing time when plotted against real time. In this section, we will focus on a well near a single boundary scenario.

As a starting point, we present a generalized analytical form of the pressure derivative (taken with respect to radial equivalent time). For simplicity, we will assume a single rate drawdown with no storage or wellbore skin:

$$\frac{\partial p}{\partial(\ln t_e)} = \frac{\partial p}{\partial(\Delta t)}\left[1+\frac{\Delta t}{t}\right]\Delta t \qquad \text{A-1}$$

Equation (A-1) states the standard equivalent-time derivative explicitly as a function of a derivative taken with respect to shut-in time. Equation (A-1) will be used to develop an analytical solution for the buildup derivative for a model depicting a well near a single boundary.

The equation for the dimensionless wellbore buildup pressure of a well near one boundary, as a function of dimensionless producing time (t_{pD}), distance to the boundary (x) and dimensionless shut-in time (Δt_D) is as follows :

$$p_D = -\frac{1}{2}Ei\left[\frac{-1}{4(t_p+\Delta t)_D}\right] + \frac{1}{2}Ei\left[\frac{-1}{4\Delta t_D}\right] - \frac{1}{2}Ei\left[\frac{-x^2}{r_w{}^2(t_p+\Delta t)_D}\right] + \frac{1}{2}Ei\left[\frac{-x^2}{r_w{}^2\Delta t_D}\right] \qquad \text{A-2a}$$

The first two terms of equation 2a represent pressure drop due to the well, while the second two terms represent the pressure drop due to an image well located a distance of 2x from the observation point. The corresponding drawdown equation for a well near a single boundary is:

$$p_D = -\frac{1}{2}Ei\left[\frac{-1}{4t_{pD}}\right] - \frac{1}{2}Ei\left[\frac{-x^2}{r_w{}^2 t_{pD}}\right] \qquad \text{A-2b}$$

Taking the derivative of equation (A-2a) with respect to real time (Δt), we get:

$$\frac{\partial p_D}{\partial \Delta t} = \frac{1}{2(t_p+\Delta t)_D}\left[-e^{\frac{-1}{4(t_p+\Delta t)_D}} - e^{\frac{-x^2}{r_w{}^2(t_p+\Delta t)_D}}\right] - \frac{1}{2\Delta t_D}\left[e^{\frac{-1}{4\Delta t_D}} + e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right] \qquad \text{A-3}$$

Substituting equation (A-3) into (A-1), we get:

$$\frac{\partial p_D}{\partial \ln(t_{eD})} = \frac{1}{2(t_p+\Delta t)_D}\left[-e^{\frac{-1}{4(t_p+\Delta t)_D}} - e^{\frac{-x^2}{r_w{}^2(t_p+\Delta t)_D}}\right] - \frac{1}{2\Delta t_D}\left[e^{\frac{-1}{4\Delta t_D}} + e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right]\left[1+\frac{\Delta t}{t_p}\right]\Delta t \qquad \text{A-4}$$

Simplifying (A-4),

$$\frac{\partial p_D}{\partial \ln(t_{e_D})} = \frac{\Delta t_D}{2t_D}\left[-e^{\frac{-1}{4(t_p+\Delta t)_D}} - e^{\frac{-x^2}{r_w{}^2(t_p+\Delta t)_D}}\right] + \frac{(t_p+\Delta t)_D}{2t_D}\left[e^{\frac{-1}{4\Delta t_D}} + e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right] \qquad \text{A-5}$$

Equation (A-5) states the equivalent time derivative explicitly as a function of dimensionless real shut-in time. The equivalent drawdown derivative for a well near a single boundary is then:

$$\frac{\partial p_D}{\partial \ln t_D} = \frac{1}{2}\left[e^{\frac{-1}{4t_{pD}}} + e^{\frac{-x^2}{r_w{}^2 t_{pD}}}\right] \qquad \text{A-6}$$

For now, at small producing times, we can make the assumption that the producing time "t_p" component in the exponential terms is insignificant compared when Δt. When we make the assumption that $\Delta t + t_p \cong \Delta t$, we get:

$$\frac{\partial p_D}{\partial \ln(t_{e_D})} = \frac{\Delta t_D}{2t_D}\left[-e^{\frac{-1}{4\Delta t_D}} - e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right] + \frac{(t_p+\Delta t)_D}{2t_D}\left[e^{\frac{-1}{4\Delta t_D}} + e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right] \qquad \text{A-7}$$

Collecting the exponential terms, we now have:

$$\frac{\partial p_D}{\partial \ln(t_{e_D})} = \left(-\frac{\Delta t_D}{2t_D} + \frac{\Delta t_D}{2t_D} + 1/2\right)\left[e^{\frac{-1}{4\Delta t_D}} + e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right] \qquad \text{A-8}$$

Taking the limit of equation (A-8) as production time approaches zero, we get:

$$\lim_{t_{p_D}\to 0}\frac{\partial p_D}{\partial \ln(t_{e_D})} = \lim_{t_{p_D}\to 0}\frac{\partial p_D}{\partial \ln(t_{e_D})}\left(-\frac{\Delta t_D}{2t_{p_D}} + \frac{\Delta t_D}{2t_{p_D}} + \frac{1}{2}\right)\left[e^{\frac{-1}{4\Delta t_D}} + e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right] = \frac{1}{2}\left[e^{\frac{-1}{4\Delta t_D}} + e^{\frac{-x^2}{r_w{}^2\Delta t_D}}\right] \qquad \text{A-9}$$

An evaluation of (A-9) will show that even at small producing times, the buildup derivative actually has the same form as the drawdown derivative (A-6) shown earlier. The only difference is that the drawdown derivative is a function of producing time "t_p", while the buildup derivative is a function real shut-in time "Δt". **Therefore, in order for to ensure that the buildup derivative maintains the same shape and character of the drawdown derivative, the buildup derivative must be plotted against real shut-in time and not equivalent time.**

If the buildup derivative is plotted against equivalent time t_e, the buildup derivative plot will be increasing compressed relative to the drawdown derivative as producing time decreases. This can be proven by taking the limit of the equivalent time function which shows that t_e will always be less than or equal to the production time t_p (refer to Figure 7)

$$\lim_{\Delta t \to \infty} te = \lim \frac{t_p \cdot \Delta t}{t_p + \Delta t} = t_p \qquad \text{A-10}$$

Thus, it is clear that the buildup derivative, when plotted against equivalent time, for any shut-in duration greater than the production time, will not show the same character as the drawdown derivative for the same reservoir system. For the sake of completeness, the limit of the buildup derivative as producing time approaches zero was re-evaluated without making the prior assumption that $\Delta t + t \cong \Delta t$ in the exponential terms. The result is shown below:

$$\begin{aligned}\lim_{t_p \to 0} \frac{\partial p_D}{\partial \ln(t_{eD})} &= \frac{1}{2}\left[e^{\frac{-1}{4\Delta t_D}} \left(1 - \frac{1}{4\Delta t_D}\right) + e^{\frac{-x^2}{r_w{}^2 \Delta t_D}} \left(1 - \frac{x^2}{r_w{}^2 \Delta t_D}\right) \right] \\ &= \frac{1}{2}\left[e^{\frac{-1}{4\Delta t_D}} f_1(\Delta t_D) + e^{\frac{-x^2}{r_w{}^2 \Delta t_D}} f_2(\Delta t_D) \right]\end{aligned} \qquad \text{A-11}$$

Analysis of equation (A-11) will show that it has the same form of as the drawdown derivative equation (A-6) given earlier, except that the exponential terms are multiplied by the functions f_1 or f_2. It is function f_1 and f_2 that were assumed to be unity in the determination of equation (A-9). It is also f_1 and f_2 that cause the slight deviation of the real buildup derivative from the drawdown derivative (as shown in Figure 8). Incidentally, it can be seen from the form of (A-11) that f_1 and f_2 will rapidly approach zero at shut-in time increases. Although the mathematical work presented in this Appendix only applies for the scenario where there is a well near a single no-flow boundary, we anticipate that similar results would be observed if the same procedure was applied to other pressure solutions.

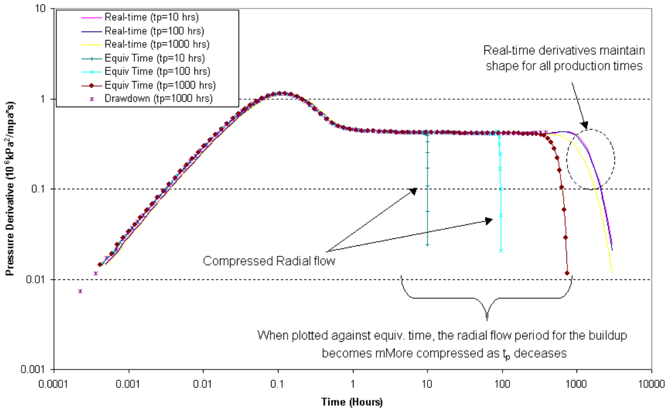

Figure 1a- Centered Well in a Closed Reservoir; Low Permeability (1 mD)

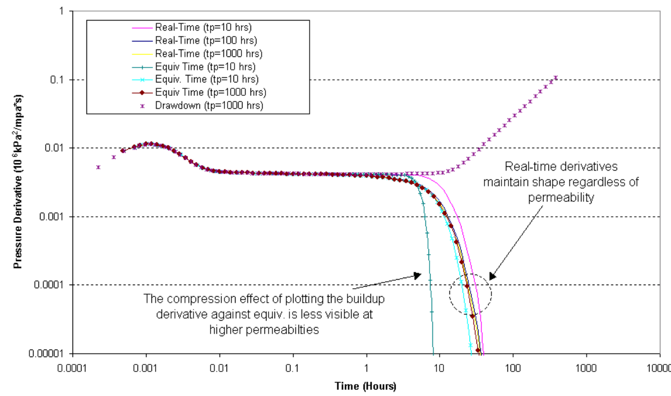

Figure 1b – Centered Well in Closed Reservoir; High Permeability (100 mD)

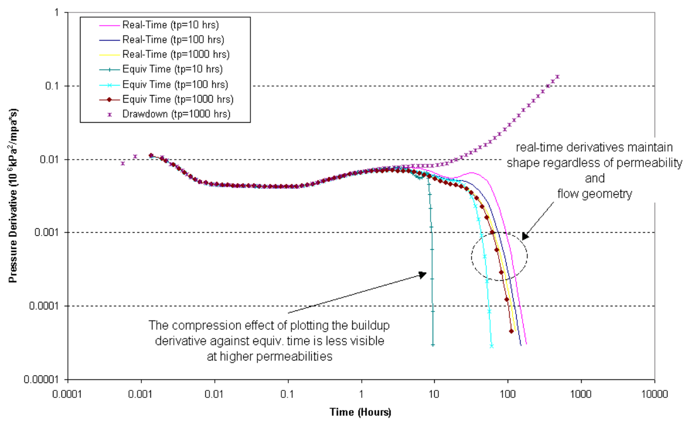

Figure 1c – Well Near One Boundary in Closed Reservoir; High Permeability (100 mD)

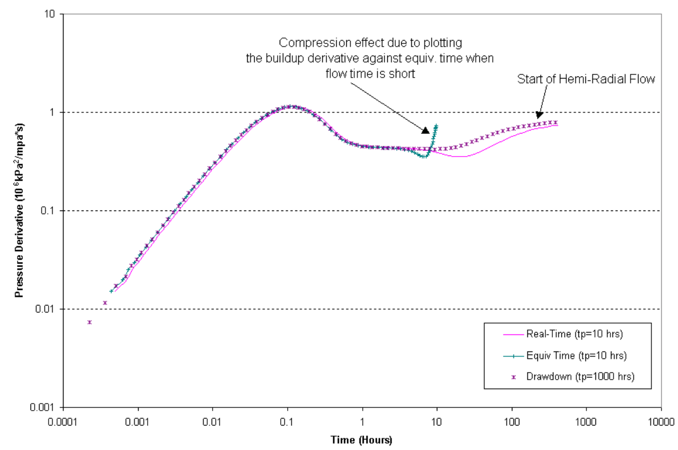

Figure 1d- Well Near One Boundary in Closed Reservoir; Low Permeability (1 mD)

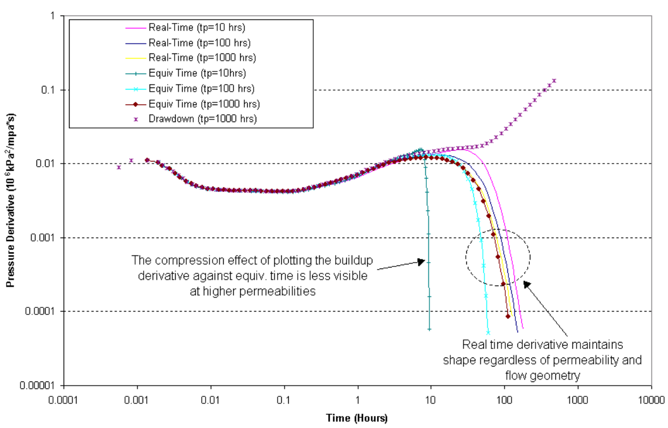

Figure 1e – Well Near Two Boundaries in a Closed Reservoir; High Permeability (100 mD)

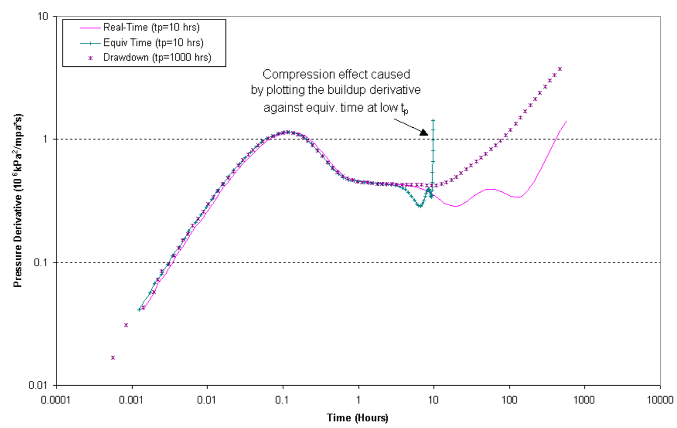

Figure 1f – Well Near Three Boundaries in a Closed Reservoir; Low Permeability (1 mD)

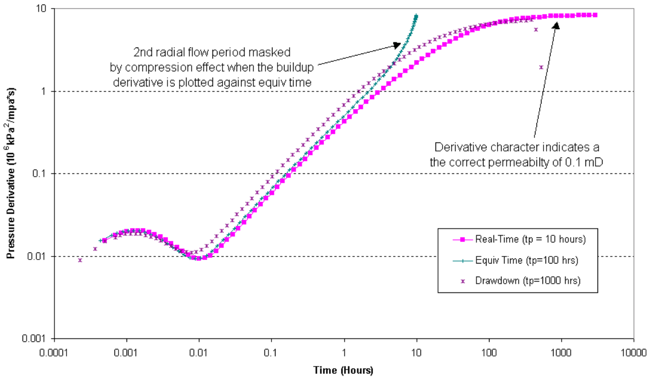

Figure 2a- High Permeability Inner Zone (100 mD); Low Permeability Outer Zone (0.1 mD)

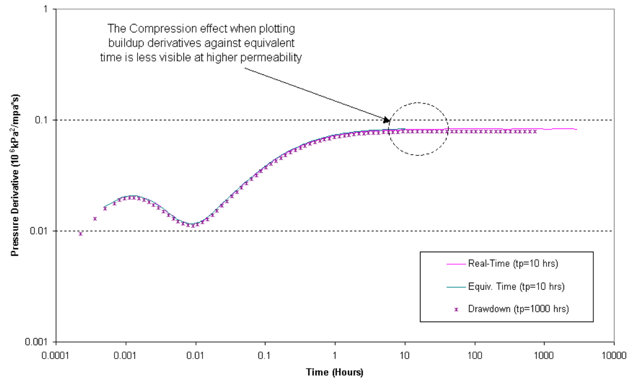

Figure 2b- High Permeabilty Inner Zone (100 mD); Low Permeabilty Outer Zone (10 mD)

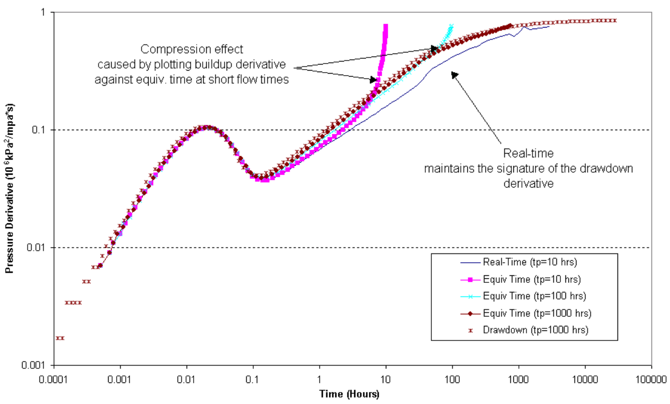

Figure 3a- Low Permeability Reservoir (1 mD, X_f=100m)

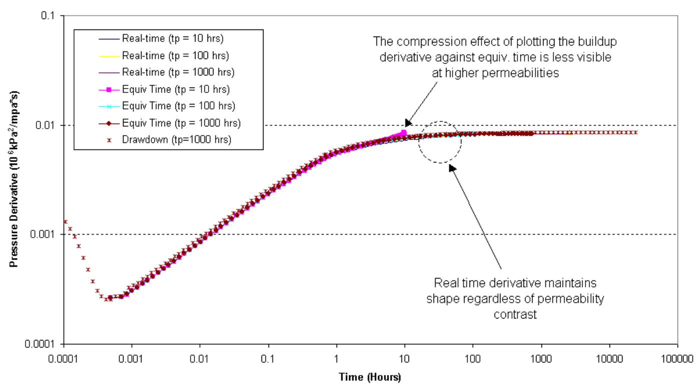

Figure 3b – High Permeability Reservoir (100 mD, X_f=100m)

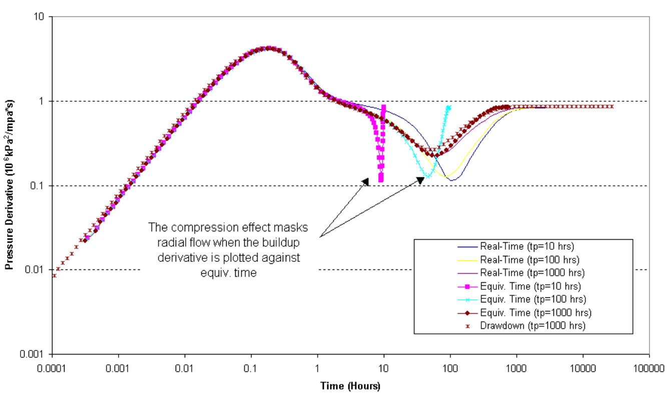

Figure 4a – Low Interporosity Flow Coefficient (matrix permeability = 1 mD, IC = 1E-6)

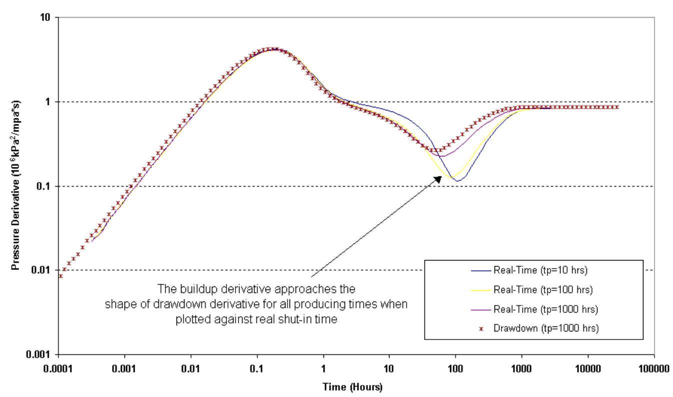

Figure 4a.2 – Low Interporosity Flow Coefficient (matrix permeability =1 mD; IC=1E-6)

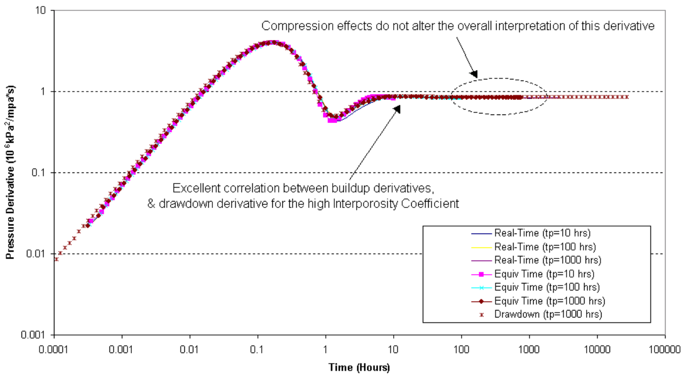

Figure 4b – High Interporosity Flow Coefficient (matrix permeability = 1 mD, IC = 1E-4)

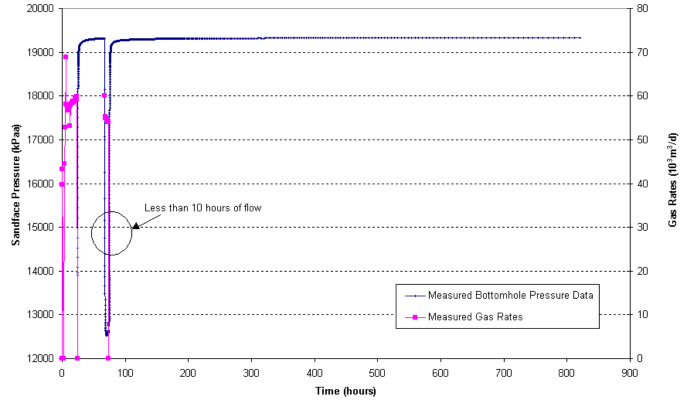

Figure 5a- Field Example #1 – Measured Rate & Pressure Data

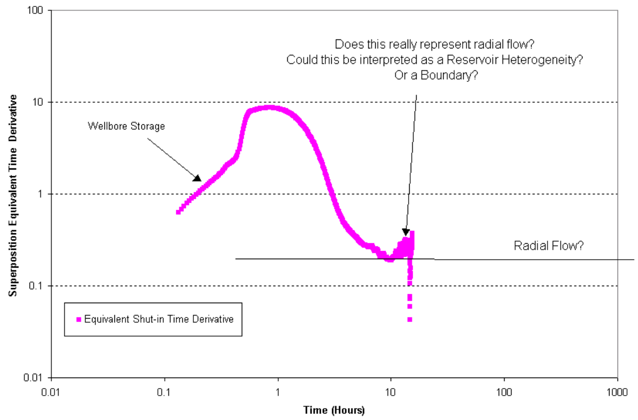

Figure 5b – Calculated Buildup Derivative (Field Example #1)

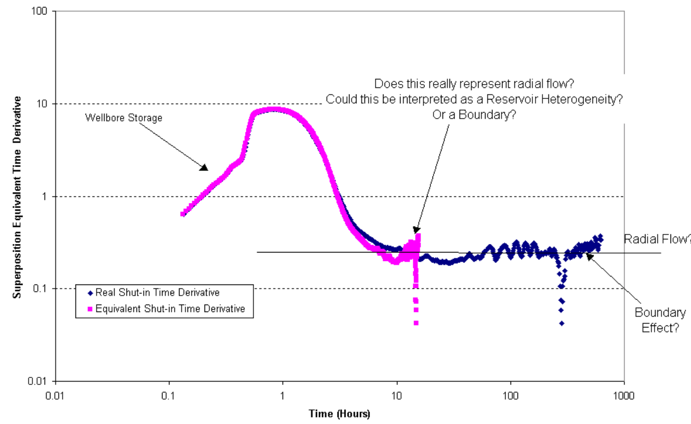

Figure 5c- Calculated Buildup Derivative (Field Example #1)

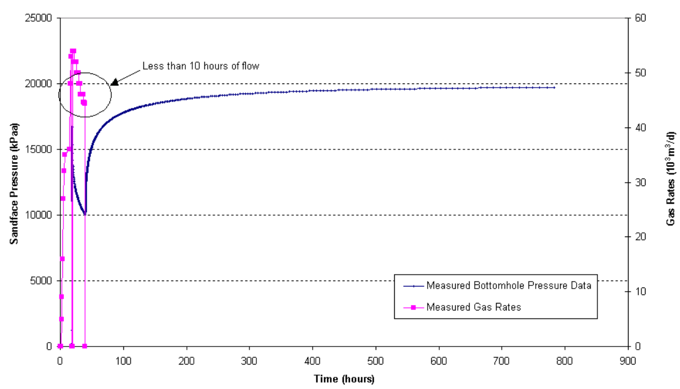

Figure 6a – Field Example #2 – Measured Rate and Pressure Data

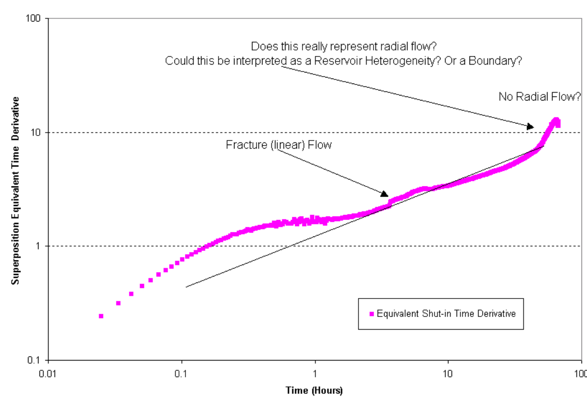

Figure 6b – Calculated Buildup Derivative (Field Example #2)

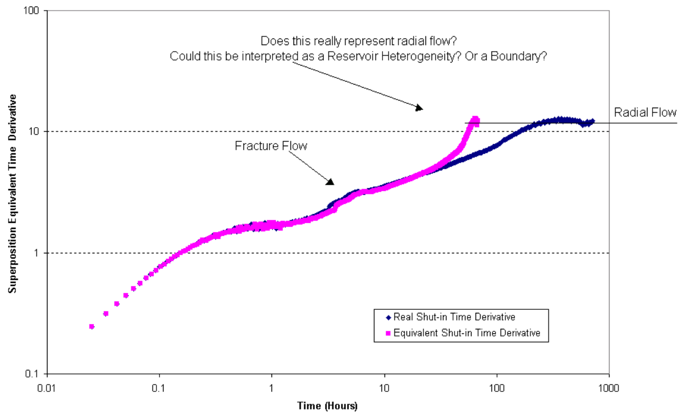

Figure 6c – Calculated Buildup Derivative (Field Example #2)

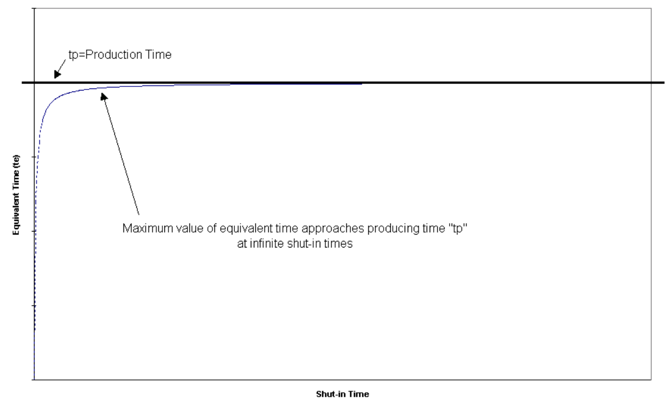

Figure 7 – Equivalent Shut-in Time as a function of Shut-in Time

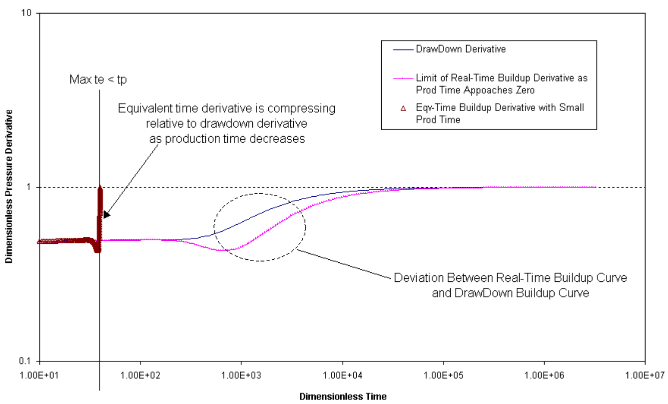

Figure 8 – Typical Derivative for a Well Near a Single No-Flow Boundary